Individual differences in general and specific cognitive precursors in early mathematical learning

Estíbaliz Aragón1, Gamal Cerda2, Cándida Delgado1, Manuel Aguilar1, and José I. Navarro1
1 University of Cádiz (Spain) and 2 University of Concepción (Chile)

Abstract

Background: The acquisition of mathematical abilities is associated not only with several academic aptitudes but also with the development of particular cognitive skills. This study analysed the role of the general and specific domain precursors of informal mathematical thinking. Method: A total of 109 4-year-old children (M = 59.30 months; SD = 3.56) participated in the study, in which the participants’ informal math and cognitive variables were assessed. A stepwise regression model was calculated. Results: The complex inferential model evidenced the role of the three general-domain variables analysed, in addition to numerical estimation as a specific-domain variable. 48.5% of participants’ variability in informal mathematical thinking, evaluated with the TEMA-3 test, was explained by three of the general domain precursors: working memory, processing speed and receptive vocabulary; as well as by estimation, a specific-domain precursor. The model showed a higher explanatory statistical weight for boys (48.9%) than girls (37.5%). Conclusions: The model indicated that working memory and processing speed were the main predictors of informal mathematical thinking at the age of four. A joint remedial or preventative intervention, taking into account predictors of the specific and general domains, could be the optimal option to improve achievement in mathematics.

Keywords: Early mathematical achievement, general-domain precursors, specific-domain precursors, mathematical cognition, gender.

Resumen

Diferencias individuales en los precursores cognitivos generales y específicos del aprendizaje matemático temprano. Antecedentes: la adquisición de habilidades matemáticas está asociada no solo con aptitudes académicas, sino también con el desarrollo de habilidades cognitivas específicas. Este estudio analizó el papel de los precursores del dominio general y específico en el pensamiento matemático informal. Método: un total de 109 niños de 4 años participaron en el estudio (M= 59.30; SD= 3.56). Se evaluaron el pensamiento matemático informal con la prueba TEMA-3, y diferentes variables cognitivas. Resultados: tras la realización de un análisis de regresión por pasos, el modelo inferencial evidenció que el 48.5% de la variabilidad de los participantes en el pensamiento matemático informal fue explicado por la memoria de trabajo, velocidad de procesamiento y vocabulario receptivo, así como por la estimación. El modelo indicó que la memoria de trabajo y la velocidad de procesamiento fueron los principales predictores del pensamiento matemático informal a la edad de cuatro años. Mostró también un mayor peso estadístico explicativo para los niños (48.9%) que para las niñas (37.5%). Conclusiones: los datos sugieren que una intervención conjunta correctiva o preventiva, teniendo en cuenta los factores predictivos de los dominios específicos y generales, podría ser la opción óptima para mejorar el rendimiento en matemáticas en niños en riesgo de tener dificultades en esta materia.

Palabras clave: aprendizaje matemático temprano, precursores de dominio general, precursores de dominio específico, cognición matemática, género.

The acquisition of mathematical abilities is associated not only with several academic aptitudes but also with the development of particular cognitive skills. Comparative studies have focused on the analysis of skills acquired in parallel with mathematical competence. These skills originate from an adequate or inadequate progress of mathematical ability (Watson, Gable, & Morin, 2016). The variables are considered to be predictors of mathematical performance, and are usually grouped into two categories: general-domain and specific-domain (Passolunghi, Lanfranchi, Altoè, & Sollazzo, 2015).

General-domain predictors refer to higher order cognitive variables that can predict the performance of several academic skills and school competencies, such as reading, writing and mathematics (Fritz, Haase, & Räsänen, 2019). Working memory (WM) and processing speed (PS) are two predictors of the general-domain. A poor WM could be responsible for low achievement in arithmetic tasks and text comprehension (Blankenship, Keith, Calkins, & Bell, 2018). Working memory (and executive functions) has been considered to be an accurate predictor of mathematical performance (Alloway & Alloway, 2010). Students who experience difficulties in learning in mathematics tend to exhibit deficits in WM (Geary, 2011). Speed of information processing involved in children’s mathematical performance is also considered to be a general-predictor for learning math (Clark, Nelson, Garza, Sheffield, Wiebe, & Espy, 2014). If the information is processed slowly, some deficiencies appear in the execution of the math task.
Individual differences in general and specific cognitive precursors in early mathematical learning

and, consequently, this could justify inadequate outcomes (Costa, Nicholson, Donlan, & Van Herwegen, 2018). These deficits in the storage and processing of information contribute to low arithmetic performance and weak numerical sense (Watson et al., 2016).

Regarding executive functions, although some authors recognize their contribution to mathematical performance, there are controversial results (McDonald & Berg, 2017; Meltzer, 2018).

Specific-domain predictors are considered to be variables that contribute to performance in a particular school skill; for example, phonological knowledge is a predictor of good reading (Ruan, Georgiou, Song, Li, & Shu, 2018), and estimation predicts mathematical abilities (Zhu, Cai, & Leung, 2017). Others examples of specific-domain predictors are skills related to the acquisition of number sense such us numerical magnitude comparison (Aragón, Navarro, Aguilar, Cerda, & García-Sedeño, 2016; De Smedt, Noël, Gilmore, & Ansari, 2013).

Students with a weak numerical sense show some severe deficits in numerical processing (Raghubar, Barnes, & Hecht, 2010). Two of the specific variables whose role in learning math still needs to be clarified are: the numerical estimation (Zhu et al., 2017), and the magnitude comparison (Sella, Lucangeli, & Zorzi, 2018).

Numerical estimation is significant because mentally represent a number on a conceptual number line (Anobile et al., 2017). Consequently, it is assumed that their ability to estimate a number line is also related to positive mathematical outcomes. However, this relationship still needs higher experimental support. The magnitude comparison has been studied as a specific-predictor of mathematical outcomes based on the relationship between symbolic and non-symbolic problems. Moreover, Xenidou-Dervou, De Smedt, van der Schoot, & van Lieshout (2013) found that they were simple at the age of 5, although there is no conclusive evidence for these results. The authors attributed the difficulty to the participants’ higher WM capacity, since the students had to store, manipulate and even trans-code the information to obtain the requested response.

Geary, Nicholas, Li, & Sun (2017) found in a longitudinal study that, in pre-school to 8th grade students, general-domain skills were more important than specific-domain skills, at least in the first years of schooling. However, as the students progressed, specific-domain predictors matched the importance of general-domain predictors. These results are similar to those obtained by Lee & Bull (2016), in another longitudinal study with comparable ages, which emphasised the importance of WM in solving arithmetic tasks.

Several studies have focused on the analysis of the predictors of math performance at different ages (Blancenship et al., 2018; Costa et al., 2018; González-Castro, Cueli, Cabeza, & Rodríguez, 2014). Their main purpose has been to find a cognitive pattern that contributes to recognizing those students who present a higher risk of mathematics learning disabilities (MLD), which may increase students’ difficulties in both their school and home lives. This detection facilitates the implementation of corrective measures with potential long-term benefits (Peake, Jiménez, & Rodríguez, 2017). However, no research has been carried out with four-year-old children.

Consequently, this study attempts to answer the scientific question of what the differential predictive capacity of: (a) general-domain precursors (verbal WM, PS and receptive vocabulary); and (b) of specific-domain (symbolic and non-symbolic comparison, and estimation), are on informal mathematical thinking tasks in 4-year-old boys and girls. Likewise, the existence of any possible differences between genders is considered.

Method

Participants

A total of 109 pre-school children participated in this study, aged between 53 and 65 months (M = 59.30, SD = 3.56). Of these, 59 were boys (54.13%, M = 59.49, SD = 3.46) and 50 girls (45.87%, M = 59.08, SD = 3.70). Students who presented special educational needs, according to expert criteria, were excluded. All children were attending one of five potential public schools. Four of them were located in Andalusia and one in Madrid, Spain. The schools had a socio-economic level corresponding to middle class standards. Students learned mathematics through the calculation based on figures procedure. This is the traditional procedure of teaching-learning of mathematics considering the main goals of the school curriculum in this matter. Consent was obtained by the principal from each of the schools and written consent by all parents/guardians of child participants in accordance with the Declaration of Helsinki Statement.

Instruments

Sub-Test of Informal Thinking Assessment. Early Mathematics Ability, TEMA-3 (Ginsburg, Baroody, del Río, & Guerra, 2007). TEMA-3’s informal thinking subtest assesses counting, comparison of quantities, informal calculation and basic informal concepts. The test was individually administered to each child and lasted between 30–40 minutes, according to the student’s age. For the current study, the informal reasoning subtest was considered, since it constitutes the main source of variability in mathematical competence at the age of 4 years (Aunin & Raisinien, 2016). In several cases, the total score of the test in children was equivalent to the numerical subtest, without the contribution of formal thinking to the score of early mathematical competence. The Cronbach’s alpha was .91.

Numerical estimation task (Siegler & Booth, 2004). This pencil-and-paper test evaluates students’ estimation in a number line. During its administration, participants are presented with a sheet of paper with a 20-centimeter number line, which starts at 0 and ends at 20. Above the line, in the upper central part of the sheet, a number is shown. Participants must point out the number in the straight line. The test consists of 10 items, which correspond to the following numbers: 2, 4, 7, 8, 11, 13, 16, 17, 18 and 19, each of which are randomly presented. The mean comparison rate was calculated according to the number of correct answers with respect to the number requested by the test versus the number provided by the student. Answers were considered to be correct if they did not present an error rate higher than +/- 15% for the requested number. The Cronbach’s alpha was .80.

Symbolic and non-symbolic Comparison Test (Nosworthy, Bugden, Archibald, Evans, & Ansari, 2013). This pencil-and-paper test evaluates students’ magnitude processing abilities. The test consists of 56 pairs of items; each is designed by two magnitudes to be compared. The figures ranged from 1 to 9. The student has to select the larger of the two magnitudes. The test runs for four minutes in total: two minutes to solve as many elements as possible for each symbolic and non-symbolic task. Both accuracy and speed were evaluated when comparing figures.

Coding subtest from the Wechsler Pre-school and Primary Scale of Intelligence, (WPPSI-III) (Wechsler, 2009). This test is
was .88.

Receptive vocabulary test from the Dyslexia Screening Test - Junior (DST-J) (Fawcett, Nicolson, Pinto, Corral, & Fernández, 2013). This test is an assessment of vocabulary mastery and reasoning ability, the purpose of which is to evaluate receptive vocabulary through a multiple-choice format. The test includes 18 items; each correct item scores one point. The participant must follow the reference models in 2 minutes. The Cronbach's alpha for this test was .84.

Backward digit task from the Dyslexia Screening Test - Junior (DST-J) (Fawcett et al., 2013). This test evaluates verbal WM. It involves the oral repetition of digits backwards. As the number of trials increases, both the number of digits and, consequently, the task difficulty increase. This task is composed of 7 series; each series comprises 2 items. The test includes 5 items; 2 of these are administered if the child has difficulties in properly understanding the instructions. The Cronbach's alpha was .88.

Procedure

Considering the sample's characteristics and the assessments used, 2 evaluation sessions were carried out by a specially trained assessment team. In the first session, participants' mathematical competence was evaluated through the TEMA-3. In the second, the general-domain mathematical performance (PS, verbal WM and receptive vocabulary) and specific-domain (estimation and comparison of magnitudes) tests were administered. The administration of the different tests was individually conducted on different school days. The evaluation ranged from between 25 to 30 minutes per session. We randomly applied both inter-sessions and intra-sessions. The assessment was carried out during school hours, and considered play-ground breaks.

Data analysis

Data analysis was carried out in a descriptive and inferential way, with the purpose of answering the research questions. Considering the inferential analyses, a stepwise multiple linear regression was carried out, using general- and specific cognitive measurements as predictive variables and informal math thinking as dependent variable. The coefficients of the independent variables in the model, as the independence of the residuals, were calculated through the Durbin-Watson D test. SPSS-24 software was used.

Results

The current study aimed to analyse the explanatory value of the variables considered to be predictors of early mathematical outcomes at the age of four. The descriptive statistics collected in Table 1 were calculated for both the informal mathematical thinking measures and the cognitive variables.

To obtain more comprehensive information about the relationship between the different variables assessed, a bivariate correlation was calculated through the Pearson coefficient (Table 2).

### Table 1

<table>
<thead>
<tr>
<th></th>
<th>Min,raw</th>
<th>Max,raw</th>
<th>Mraw (SD)</th>
<th>Mboys (SD)</th>
<th>Mgirls (SD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Receptive vocabulary</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Processing speed</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Verbal working memory</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Symbolic comparison</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-symbolic calculation</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Estimation</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Informal thinking</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Numeration</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Comparison</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Informal calculation</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Informal concepts</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A stepwise multiple linear regression analysis was calculated, and four models were established (Table 3). Data meet the three multiple linear regression assumptions: the relationship between independent and dependent variables was linear; errors between observed and predicted values were normally distributed; and no multicollinearity was found. The fourth model showed the highest explanatory capability. Consequently, and considering the corrected R² data, a 48.5% variance in students' informal thinking could be predicted by verbal WM, estimation, PS and receptive vocabulary. Likewise, the t value was associated with an error probability of lower than .05 (p < .05) in the four variables included in the predictive model. The results of the t test and its critical values contributed to the contrasting null hypothesis: the resulting regression coefficient was zero.

On the other hand, the standardised coefficients (Table 4) demonstrated evidence for the statistical weight of each variable introduced in the explanatory model represented in the explanation of the dependent variable: verbal WM (β = .444); estimation (β = .253); PS (β = .254); and receptive vocabulary (β = .152). All values contributed either favourably or incrementally to explaining the variability of the informal math thinking scores.

The comparison of the magnitude variables with Arabic numbers (β = .080; t = 1.00, p > .05) and with points (β = -.102; t = -1.37; p > .05) were excluded from the model.
Regarding the model's validity, the independence of the residual values was calculated through the Durbin-Watson $d$ statistic ($d = 1.83$). This value, being so close to 2, confirmed the absence of autocorrelation. Similarly, the absence of multicollinearity and the stability of estimations was also assumed when obtaining acceptable Tolerance and VIF values.

In order to examine whether gender in some way affected the resulting model in the variables, the percentage of variability explained and their relatively statistic weights, the sample was distributed according to gender. Thereafter, a different stepwise multiple linear regression was calculated (Table 5).

After the comparative analysis, it can be assumed that the variables incorporated into the model had a higher explanatory power of the variability of informal mathematical thinking for boys; this analysis explained 48.9% of the variance. However, for girls this value only reached 37.1%. The models for boys and girls agreed on the variables that had the highest statistical weight in the explanation of informal thinking. WM was the variable with the highest weight, both for boys ($\beta = .495; t = 5.15, p < .01$) and girls ($\beta = .537; t = 4.73, p < .05$). The following value was estimated for both boys ($\beta = .321; t = 3.37; p < .01$) and girls ($\beta = .313; t = 2.76; p < .01$). For boys, the PS was the third variable introduced in the model ($\beta = .256; t = 2.64; p < .01$).

The Durbin-Watson $d$ statistic, with values near 2, guaranteed the independence of the residues in both models. In the same way, stability was observed in the estimations and collinearity absence as a function of the values of Tolerance ($T$) and Variance Inflation Factor ($VIF$) values.
Explanatory models of early year’s mathematical performance attempt to understand what specific and general factors support good mathematical outcomes. These models can provide useful information for early intervention programmes for students at risk of developing MLD (Fletcher, Lyon, Fuchs, & Barnes, 2018). In this study, both variables were considered to be general-domain predictors of early math skills (WM, PS and receptive vocabulary), as well as evaluating specific-domains (estimation and magnitudes comparison). Furthermore, informal math skills were also evaluated. A resulting complex inferential model evidenced the role of both the three general-domain variables analysed and the numerical estimation as a specific-domain variable.

Regarding the general-domain predictors, the model highlighted WM as the main predictor of informal mathematical thinking at the age of four; these results are consistent with recent literature (Lee & Bull, 2016; McDonald & Berg, 2017). This suggests that WM had a significant role in solving tasks of numeration, comparison, informal calculation, and informal concepts. In the first few years of schooling, WM is convenient for multiple tasks related to mathematical achievement (comparison, informal calculation): maintaining information when calculating or understanding a problem, accessing information stored in the long-term memory, the recovery of numerical facts and in problem representation (Hubber, Gilmore, & Cragg, 2014). High levels of WM should contribute to efficient task performance; a prerequisite of mathematical achievement. However, the weakening of these abilities will entail difficulty when meeting mathematical requirements. Consequently, it seems that WM is important in the first years of schooling, but its impact on academic achievement is reduced as the student reaches adolescence and adulthood (Chu, van Marle, & Geary, 2016).

Students at risk of MLD may lack an efficient WM; this generates problems in the recovery of content, and is linked to numerical PS (Peng & Fuchs, 2016). A higher PS should increase the amount of information that can be analysed at any one time. Therefore, it allows the brain to keep the information and avoid losing it. This increases the probability of simultaneously carrying out several tasks. Consequently, it follows that the PS is involved in the efficiency of WM’s central executive, especially at an early age. According to some well-established studies (Clark et al., 2014), little by little the influence of the central executive is split, acquiring an independent importance in the explanation of mathematical performance.

In the same way, this study seems confirm the role of PS as a relevant variable in explaining numerical task performance, quantity comparison, informal calculation and basic informal mathematical concepts. These results coincide with those of other authors, revealing the role of PS in solving mathematical tasks (Clark et al., 2014; Geary, 2011). Students processing information more slowly experience increased difficulties in successfully solving a mathematical task (Costa et al., 2018).

Within the suggested model, the positive role of the receptive vocabulary was also established, in agreement with others recent studies (Bleses, Makransky, Dale, Højen, & Ari, 2016; Chow & Ekhholm, 2019). Receptive vocabulary predicted higher performance in early mathematics than executive functions (Harvey & Miller, 2017). Consequently, children’s fluency in reciting the counting chain, their understanding of words’ cardinal numeric value and the recognition of Arabic numerals are positively associated with long-term mathematical achievement (Geary & van Marle, 2016). On the other hand, in this study the regression analysis excluded the magnitude comparison as a specific-domain predictor of informal mathematical thinking at the age of 4. The statistical inferential analysis also excluded the symbolic comparisons (symbolic and non-symbolic) as domain-specific predictors of informal mathematical skills. Although the symbolic comparison correlated with informal thinking, it was not so in the case of the non-symbolic comparison; in neither case were the comparison variables included in the explanatory model. However, numerical estimation was included with high predictive influence. The ability to adequately estimate implies being aware of numerosity and its distribution on a number-line oriented from left to right. It is also closely related to the Approximate Number System (ANS). This system is responsible for representing approximate magnitudes. These would be placed in ascending order on the number line (from left to right depending on the number), while the numbers would have their corresponding place in the spatial representation (Previtali, de Hevia, & Girelli, 2010). This makes sense, given that children with MLD appear to have a deficit in ANS. They have difficulties in performing adequate estimation tasks, showing high values in Weber’s fractions (Mazzocco, Feigenson, & Halberda, 2011). Other authors are of the view that this deficit is due to the disconnection between the symbolic representation and the notion of the innate magnitude (Rousselle & Noël, 2007). This discrepancy may be due to the fact that the ANS operates on multiple modalities of information, thus establishing imprecise representations. It seems that at the age of four, estimation skills should have a significant responsibility in the explanation of mathematical performance, being the one specific-domain variable included in our model. The literature provides evidence of the relationship between symbolic numerical comparison and struggles in mathematics; this is possibly due to a lack of access to meaning. These results have been found in first grade (De Smedt & Gilmore, 2011), second (Rousselle & Noël, 2007) and third grade of primary education (Iuculano, Tang, Hall, & Butterworth, 2008). However, we did not find this relationship in the comparison of non-symbolic magnitude (for a review, see De Smedt et al., 2013). Our results suggest that magnitude comparison (symbolic and non-symbolic) does not explain mathematics outcomes, particularly considering students with different math achievement levels. However, there was a significant correlation between symbolic comparison and informal mathematical thinking at the age of 4. Although it is true that, in higher numbers, the studies found the prediction effect for the symbolic comparison, some did not (De Smedt et al., 2013; Ferreira et al., 2012). In addition, this predictive effect decreased as age advanced (Sasanguie, De Smedt, Deteye, & Reynvoet, 2012). Ferreira et al. (2012) suggested that students with high and low math achievements can access symbolic and non-symbolic representations, but those who show MLD are less accurate in non-symbolic tasks. Consequently, it seems that the ANS is a more reliable predictor than the cognitive procedure for analysing the numerical magnitude in a more precise way. This would compensate deficits in ANS (Halberda & Feigenson, 2008).

Finally, a math performance distinction was made for gender. The results showed that the model has a higher predictive power for boys than girls, but that no significant differences were found when comparing performance in informal mathematical reasoning tasks. A noteworthy difference between the two models
was that the model calculated for boys incorporated the PS as a significant variable; this could indicate some type of differential or complementary cognitive activity that deserves to be examined a posteriori. Math performance descriptive data showed significant differences for girls (Palejwala & Fine, 2015), but the explanatory weight of this variable on informal thinking was significant for boys alone.

There are two potential limitations on this study. First, the nature of the research design could be considered a limitation of the study; this did not include follow-up monitoring measures in the form of a longitudinal study. This kind of design could support the cumulative or consistent effect of the role of cognitive variables (such us IQ) on the examined mathematics tasks and their outcomes. Eventually, longitudinal studies could also demonstrate their effect on subsequent students' academic performance, improving the generalization of the study as well as the practical implications. These possible effects should confirm their role as a precursor. A second limitation could refer to the restrictive characteristics of the sample, both in age and the type of school establishment, without including variables such as teaching methodological approaches, which could affect the results differentially. This study allowed us to analyse the differential role of precursors in an exploratory rather than a causal way.

Further longitudinal studies, as well as the analysis of the impact of early interventions of this type of cognitive precursor through multimedia support, are important future directions in this field. This could reduce mathematics learning difficulties and its chronicization in students with low performance in the cognitive precursor variables associated with low mathematical achievements.

An early intervention would increase the mathematical performance of students that frequently show achievement difficulties. Future research will need to follow the use of these remedial intervention programmes and continue to provide structure and support the role of cognitive precursors on the mathematical outcomes in children at risk of mathematics learning difficulties.

In conclusion, a joint remedial or preventative intervention, taking into account the specific- and general-domain predictors, could be the optimal option to improve achievement in mathematics, particularly for students at risk of MLD (Hornung, Schiltz, Brunner, & Martin, 2014). Similarly, teaching isolated general cognitive skills such as WM does not seem to successfully contribute to improving academic achievement in math (Raghubar et al., 2010). In short, there is a clear need to consider both types of predictors when implementing preventative intervention programmes for students at risk of presenting math learning disabilities.

Conflict of interests

The authors declare that they have no conflict of interests relating to this article.

Acknowledgements

This work was supported by the MINECO/FEDER Spanish government project PSI2015-63856-P; FONDECYT n. 1191064; FONDECYT n. 1160980 and CONICYT Basal Funding Program (FB0003) Chilean government.

References


